Theoretical Economics

The Competitive Stock Market as Cartel Maker: Some Examples

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1. Introduction

When will a firm have reason to purchase a share of the ownership of a competitor? In many cases, such purchases have to do with an attempt by one firm to take over -- or to gain a measure of control over -- a competitor. However, there are also cases in which small but significant fractions of a firm's ownership are held by its competitors, without control being at issue in any way. This is true, for example, in the Israeli banking industry, where significant fractions of the ownership of commercial banks are held, often indirectly, by other commercial banks. Such behaviour cannot be explained by a firm's desire to diversify, because the earnings of firms in the same industry are, in most cases, very highly correlated. Our purpose here is to suggest another explanation.

Suppose that we have several firms competing with one another in an industry. Suppose also that cooperation or collusion are impossible,

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either because of effective anti-trust measures or, more generally, because collusive contracts are not enforceable. To keep matters simple, let us think of the firms as family businesses, where management and ownership coincide. At some point, these firms approach the Securities and Exchange Commission, applying for permission to have shares of their ownership traded in the stock market. Should a Securities and Exchange Commission, whose purpose is to protect the public against cartelization, grant this application? Our own first answer would have been that if the market in which the shares are to be traded is strictly competitive, then there should be no reason to deny the application. This answer is wrong, as the examples below will show. It is quite possible for firms who would like to collude but cannot do so to realize the benefits of collusion by trading shares in a perfectly competitive stock market. In other words, it is possible for a competitive stock market to act as cartel-maker.

Competitive stock markets have been studied in considerable detail recently (see, for example, Grossman and Hart [1979]). In all of these studies, so far as our reading goes, it is assumed that only consumers can hold shares of stock. Here, our aim is to suggest that the case where one firm may own a share in another firm is also of interest.

2. A Definition

Let $G = \langle X, Y, u^1, u^2 \rangle$ be a two-player game in normal form, with X and Y being the respective strategy sets of the players and u^1, u^2 their respective payoff functions. Both u^1 and u^2 are real-valued functions

defined on X x Y. We shall think of the values of u^1 and u^2 as denominated in monetary units, say dollars, although other objectively measurable numerical units (such as years or imprisonment in the prisoners' dilemma) are also possible. We can think of the players' roles in the game G as consisting of ownership and control: Player i owns the payoff machine u^i and his control consists of selecting a strategy from the appropriate set, X or Y. At some point, shares of the players' ownerships become tradable in a competitive stock market, with the trades in ownership having no effect whatsoever on the players' control. Thus, in principle, a player could acquire 100% ownership in another player's payoff, with this other player nevertheless retaining full control. This possibility will not arise, however, in the present discussion. In our examples, it is never required, in equilibrium, that a player should hold even a majority (to say nothing of holding 100%) of another player's shares. The objects which are to be traded in the market are shares in the players' ownerships of the respective payoff functions, not shares in the players' total ex-post payoffs. Each payoff function should be thought of as a payoff-producing machine, with the players trading shares of ownership in these machines.

Let player 1's demand vector be denoted $\underset{\sim}{d} = (d_1, d_2)$, with both d_1 and d_2 lying in the unit interval. d_1 is player 1's demand for shares in u^1 and d_2 is his demand for shares in u^2. Define the game $G(\underset{\sim}{d})$ by writing

$$G(\underset{\sim}{d}) = \langle X, Y, \underset{\sim}{d} \cdot \underset{\sim}{u}, (\mathbb{1} - \underset{\sim}{d}) \cdot \underset{\sim}{u} \rangle$$

where $\underset{\sim}{u} = (u^1, u^2)$, $\mathbb{1} = (1,1)$, and $\cdot$ is scalar multiplication.

When player 1 contemplates the demand vector $\underset{\sim}{d}$, he does so in the expectation that, if this demand vector were to be realized, the game which would be played would be $G(\underset{\sim}{d})$. Thus, player 1 always assumes that whatever shares are not held by himself are held by his opponent, player 2. Similarly, suppose that player 2 contemplates the demand vector $\underset{\sim}{d}' = (d_1', d_2')$, which would put him in possession of a fraction d_1' of u^1 and a fraction d_2' of u^2. From player 2's point of view, the demand vector $\underset{\sim}{d}'$ is evaluated on the expectation that the game to be played after trading is completed would be $G(\amalg - \underset{\sim}{d}')$. We shall insist, of course, that players' expectations be fulfilled in equilibrium.

For every $\underset{\sim}{d} \in [0,1]^2$, we define $N(\underset{\sim}{d})$ by writing

$$N(\underset{\sim}{d}) = \left\{ (x,y) \in X \times Y \;\middle|\; \begin{array}{l}(x,y) \text{ is a Nash} \\ \text{equilibrium of } G(\underset{\sim}{d})\end{array} \right\} .$$

We shall refer to $N(\cdot)$ as the <u>Nash correspondence</u> associated with the given game G. A <u>selection</u> from the Nash correspondence N is a pair $(\hat{x},\hat{y})$ of functions, $\hat{x} : [0,1]^2 \to X$ and $\hat{y} : [0,1]^2 \to Y$, such that $(\hat{x}(\underset{\sim}{d}),\hat{y}(\underset{\sim}{d})) \in N(\underset{\sim}{d})$ for every $\underset{\sim}{d} \in [0,1]^2$.

Given a selection $(\hat{x},\hat{y})$ from the Nash correspondence N, we can define two "utility" functions, U and V, by

$$U(\underset{\sim}{d}) = \underset{\sim}{d} \cdot \underset{\sim}{u}(\hat{x}(\underset{\sim}{d}),\hat{y}(\underset{\sim}{d}))$$

$$V(\underset{\sim}{d}) = \underset{\sim}{d} \cdot \underset{\sim}{u}(\hat{x}(\amalg - \underset{\sim}{d}),\hat{y}(\amalg - \underset{\sim}{d})),$$

for all $\underset{\sim}{d} \in [0,1]^2$. $U(\underset{\sim}{d})$ and $V(\underset{\sim}{d})$ are the respective payoffs of player 1 and player 2 at asset position $\underset{\sim}{d}$. That is, if $\underset{\sim}{d} = (d_1,d_2)$, then $U(\underset{\sim}{d})$ is the total ex-post payoff which player 1 expects to receive when holding a share d_1 of u^1 and a share d_2 of u^2. Similarly, $V(\underset{\sim}{d})$ is the total ex-post payoff which player 2 expects to receive when holding d^1 in u^1 and d^2 in u^2. We shall bear in mind that both U and V are defined relative to some specific selection, $(\hat{x},\hat{y})$, from the Nash correspondence N.

The trading of shares is obviously just a vehicle for redistributing the overall payoff to the two players. That is, for each $\underset{\sim}{d} \in [0,1]^2$, we have,

$$U(\underset{\sim}{d}) + V(\amalg - \underset{\sim}{d}) = u^1(\hat{x}(\underset{\sim}{d}),\hat{y}(\underset{\sim}{d})) + u^2(\hat{x}(\underset{\sim}{d}),\hat{y}(\underset{\sim}{d})).$$

In particular, if $\underset{\sim}{d} = (\frac{1}{2},\frac{1}{2})$ then both players' post-trade payoffs coincide and it is not surprising, in this case, that the players are free to get on with the business of securing a joint maximum. From this observation, we are led to the following two questions:

a. Will such a joint maximum be supported by a competitive equilibrium in the stock market?

b. Will it be necessary for each player to hold a full half of the shares of the other player's payoff function in order to obtain the joint maximum?

Our hope is to shed some light on the answers to precisely these two questions.

<u>DEFINITION</u>. Let p and q be nonnegative real numbers, and let $\underset{\sim}{d}^1$ and $\underset{\sim}{d}^2$ be two vectors in the unit square, $[0,1]^2$. We shall say that $\langle p,q,\underset{\sim}{d}^1,\underset{\sim}{d}^2 \rangle$ is a <u>competitive stock market equilibrium</u> (for the game G) if there exists a selection $(\hat{x},\hat{y})$ from the Nash correspondence N, such that

(1) $\underset{\sim}{d}^1$ maximizes $U(\underset{\sim}{d})$ under $\underset{\sim}{d}\cdot(p,q) = p$

(2) $\underset{\sim}{d}^2$ maximizes $V(\underset{\sim}{d})$ under $\underset{\sim}{d}\cdot(p,q) = q$

(3) $\underset{\sim}{d}^1 + \underset{\sim}{d}^2 = \mathbb{1}$.

Competitive equilibrium appears in this definition in its classical Walrasian (or "Edgeworth box") sense, with an auctioneer or some similar artifact being used to guarantee price-taking behaviour. Thus, (1) and (2) require that the players' demand vectors be utility-maximizing in the respective budget sets, given the initial endowments (1,0) and (0,1). Condition (3) is the equality of supply and demand. Both players' consumption sets consist of the unit square.

With only two agents in the market, it is of course unlikely for a Walrasian trade structure to emerge, and we are not suggesting that it would. Rather, we ask how the two players would behave if the only available avenue for trade was the two-trader Walrasian market, unlikely as the emergence of this kind of market might be. Surely, no one would question the "competitiveness" of such a market, so we use it in our definition as a paradigm for competition.

At a competitive stock market equilibrium, the players' payoffs are always at least as high as their respective payoffs at the appropriate Nash equilibrium of the original game G. Formally, we have:

<u>ASSERTION</u>. Let $\langle p,q,\underset{\sim}{d}^1,\underset{\sim}{d}^2\rangle$ be a competitive stock market equilibrium, and let the pair (1,0) be denoted $\underset{\sim}{e}$. Then, $U(\underset{\sim}{d}^1) \geq u^1(\hat{x}(\underset{\sim}{e}),\hat{y}(\underset{\sim}{e}))$ and

$V(\underset{\sim}{d}^2) \geq u^2(\hat{x}(\underset{\sim}{e}),\hat{y}(\underset{\sim}{e}))$.

This assertion follows immediately from the Pareto-efficiency of competitive equilibrium.

A competitive stock market equilibrium, as defined here, is preserved when both players' payyoff functions are replaced by the <u>same</u> monotone transform of themselves. It is not preserved, however, when the monotone transformations being applied are different for the two payoff functions. In other words, under our definition, a competitive stock market equilibrium is a co-ordinal concept, but not an ordinal one. Another property about which one may wish to inquire is homogeneity: Will a doubling of player 1's payoff function lead to a doubling of the price of player 1's shares at a competitive stock market equilibrium? More precisely, we can state the question as follows: Let $\langle p,q,\underset{\sim}{d}^1,\underset{\sim}{d}^2\rangle$ be a competitive stock market equilibrium for $G = \langle X,Y,u^1,u^2\rangle$ and let α and β be two positive real numbers. Does there exist a competitive stock market equilibrium for the game $G' = \langle X,Y,\alpha u^1,\beta u^2\rangle$ with prices αp and βq? The answer to this question is no. (See Remark, following Example 2, below.)

3. Examples

<u>Example 1</u>. Let a game G_1 be given by

G_1 :

T	3,0
B	2,2

with player 1 picking a row and player 2 having no strategic choice whatsoever. Will trading in a competitive stock market make it possible for player 1 to switch to his second strategy (marked "B") and play the joint maximum? The answer is no. In fact, for this game G_1, a competitive stock market equilibrium does not exist. [Proof: Let $\langle p, q, \underset{\sim}{d}^1, \underset{\sim}{d}^2 \rangle$ be a competitive stock market equilibrium for G_1, and write $\underset{\sim}{d}^1 = \underset{\sim}{d}$, $\underset{\sim}{d}^2 = \mathbb{1} - \underset{\sim}{d}$. We will show that $\underset{\sim}{d}$ cannot be a utility-maximizing demand vector for player 1. Following our previous notation, we write $\hat{x}(\underset{\sim}{d})$ for player 1's optimal strategy in the game $G_1(\underset{\sim}{d})$. Suppose first that $\hat{x}(\underset{\sim}{d}) = T$. Then we must have $q = 0$, because at $q > 0$ there would be a positive excess supply of player 2's shares. Also, if $\hat{x}(\underset{\sim}{d}) = T$ then $U(\underset{\sim}{d}) \leqq 3$. Since $q = 0$, player 1 can afford to purchase the demand vector $d' = (1,1)$. However, $U(\underset{\sim}{d}') = 4$, and $\underset{\sim}{d}$ is not utility-maximizing. Now suppose that $\hat{x}(\underset{\sim}{d}) = B$. Then, we must have $p > 0$, $q > 0$, and $p = q$. (If $p \neq q$, then supply and demand cannot be equal.) Player 1's budget constraint now takes the form $d_1 + d_2 \leqq 1$, from which it follows that $U(\underset{\sim}{d}) = 2d_1 + 2d_2 \leqq 2$. But the vector $\underset{\sim}{d}'' = (1,0)$ is player 1's initial endowment, so it certainly lies within his budget, and for $\underset{\sim}{d}''$ we have $U(\underset{\sim}{d}'') = 3$. Once again, $\underset{\sim}{d}$ is not utility-maximizing.] Thus we see that, even for very simple-minded games, the joint maximum may not be supportable by a competitive stock market equilibrium.

Remark. Non-existence in this example has nothing to do with the restriction to pure strategies. Indeed, if we replace the game G_1 by its mixed extension, we still get the result that a competitive stock market equilibrium fails to exist.

Example 2. Let us modify the game of the previous example, and consider

G_2:

	L	R
T	3,0	1,1
B	2,2	1,1

with player 1 picking a row and player 2 picking a column. In G_2, player 2 can choose to play the previous game, G_1. If he does not, then both players automatically earn a payoff of one unit each.

For this game, G_2, a competitive stock market equilibrium exists. Letting $p = 1$, $q = 1$, and $\underset{\sim}{d} = (2/3, 1/3)$, we find that $\langle p, q, \underset{\sim}{d}, \mathbb{1} - \underset{\sim}{d} \rangle$ is a competitive stock market equilibrium for G_2. This equilibrium is not unique, but most of its features are. To be more precise, we state the following assertion: $\langle p, q, \underset{\sim}{d}, \mathbb{1} - \underset{\sim}{d} \rangle$ is a competitive stock market equilibrium for G_2 if, and only if, $p = q \neq 0$ and $\underset{\sim}{d} = (d_1, d_2)$ satisfies $d_1 + d_2 = 1$, $d_1 \leqq 2/3$. Moreover, in all of these equilibria, $\hat{x}(\underset{\sim}{d}) = B$, $\hat{y}(\underset{\sim}{d}) = L$, and $U(\underset{\sim}{d}) = V(\mathbb{1} - \underset{\sim}{d}) = 2$, i.e., the joint maximum is being played and is being split equally between the two players. (The proof of this assertion is omitted. It consists of calculations which are straightforward but tedious.) In this example, the fraction of the opponent's shares held by each player at equilibrium is at least 1/3. The stock market allows the players to achieve a joint maximum without requiring that ownerships be shared equally.

Remark. It might be of interest to check what happens when, in the game G_2, we double the payoffs of one player, say player 1. Let us therefore consider the game G_2', given by

G_2' :

6,0	2,1
4,2	2,1

where, as before, player 1 picks a row and player 2 picks a column. Here, it turns out that a competitive stock market equilibrium (for the game G_2') no longer exists. If, on the other hand, we double player 2's payoffs (rather than player 1's) in the original game G_2, then a competitive stock market equilibrium does exist, with the price of player 2's shares doubled. That is, if we consider the game

G_2'' :

3,0	1,2
2,4	1,2

then we find that a competitive stock market equilibrium for G_2'' does exist. In particular, if $q = 2p > 0$ and $\underset{\sim}{d}^1 = (2/3,1/6)$ $\underset{\sim}{d}^2 = (1/3,5/6)$, then $\langle p,q,\underset{\sim}{d}^1,\underset{\sim}{d}^2\rangle$ is a competitive stock market equilibrium for G_2''. The players' payoffs at all competitive stock market equilibria for G_2'' are given by 2 for player 1 and 4 for player 2.

Example 3. A slight generalization of the previous example will give rise to a game for which a competitve stock market equilibrium exists, with the property that share prices are equal to any arbitrary pre-assigned values. Let $p \geqq 0$ and $q > 0$ be given, and write $p/q = s$. For some $k \geqq 1$, consider the game G_3, given by

G_3 :

ks + 1, 0	s, 1
ks, k	s, 1

where, as usual, player 1 picks a row and player 2 picks a column. (For $k = 2$ and $s = 1$, we get the game G_2 of the previous example.) If, given G_3, we allow the players to trade shares in a competitive stock market, we find that there exists a competitive stock market equilibrium $\langle p,q,\underset{\sim}{d}^1,\underset{\sim}{d}^2\rangle$ for which p and q coincide with their pre-assigned values. Furthermore, if we take $\underset{\sim}{d}^1$ and $\underset{\sim}{d}^2$ to be the final share holdings of the two players at a minimum-trade equilibrium, we find that

$\underset{\sim}{d}^1 = (\frac{ks}{ks+1}, \frac{s}{ks+1})$ and $\underset{\sim}{d}^2 = (\frac{1}{ks+1}, \frac{(k-1)s+1}{ks+1})$. That is, the least amount of player 2's shares which player 1 holds at equilibrium is $s/(ks + 1)$ and the least amount of 1's shares held by 2 at equilibrium is $1/(ks + 1)$. For large k, both of these quantities tend to 0, so that holding a very small fraction of the opponent's shares suffices to secure the joint maximum at a competitive stock market equilibrium. It should be noted that all the competitive stock market equilibria for G_3 are joint-maximum equilibria, and the distribution of the final payoff in all of them is given by ks for player 1 and k for player 2.

Example 4. It may be of some interest to see what happens when a competitive stock market is introduced into the framework of a classical prisoners' dilemma. Let α and β be two real numbers satisfying $0 < \alpha < \beta < 1$, and consider the game G_4 given by

G_4 :

β,β	0,1
1,0	α,α

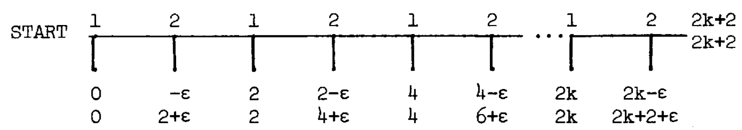
An analysis of a competitive stock market for this game reveals the following two cases: Case a: $\alpha + \beta \geqq 1$. When this inequality holds, a competitive stock market equilibrium always exists. Indeed,

$\langle p,q,\underset{\sim}{d},\Pi - \underset{\sim}{d}\rangle$ is a competitive stock market equilibrium in this case if, and only if, $p = q$ and $\underset{\sim}{d} = (d_1,d_2)$ satisfies $d_1 + d_2 = 1$, $d_2 \geqq 1 - \beta$. Both players play co-operatively. At a <u>minimum trade</u> equilibrium, each player holds a fraction $1 - \beta$ of his opponent's shares. (Note that, in this present case, we always have $1 - \beta < 1/2$.)

<u>Case b</u>: $\alpha + \beta < 1$. Here it turns out that a competitive stock market equilibrium <u>no longer exists</u>. In other words, when $\alpha + \beta < 1$, the competitive stock market is no longer capable of acting as a vehicle for compromise.

The condition for existence of a competitive stock market equilibrium for G_4, namely $\alpha + \beta \geqq 1$, can be interpreted as saying that the payoff to a player who defects from co-operation should not be too large, in comparison with the other payoffs in the game.

<u>Example 5</u>. (Rosenthal's game). In a recent paper [1981], Robert Rosenthal presented a game in extensive form where Nash behaviour seemed rather unconvincing. For $\varepsilon > 0$, the following is a version of Rosenthal's game:



In this game, the players' payoffs at a Nash equilibrium are the pair (0,0), with player 1 playing "down" at the first move. Define the sets X and Y and the functions u^1 and u^2 by writing -

$$X = Y = \{0,1, \ldots k + 1\}$$

$$u^1(x,y) = \begin{cases} 2x & \text{if } x \leqq y \\ 2y - \varepsilon & \text{if } x > y \end{cases}$$

$$u^2(x,y) = \begin{cases} 2x & \text{if } x \leqq y \\ 2y + 2 + \varepsilon & \text{if } x > y \end{cases}$$

for $(x,y) \in X \times Y$. $G_5 = \langle X,Y,u^1,u^2\rangle$ is a normal-form game corresponding to Rosenthal's game, and we can proceed to ask about a competitive stock market equilibrium for G_5. What we find is that $\langle p,q,\underset{\sim}{d},\Pi - \underset{\sim}{d}\rangle$ is a competitive stock market equilibrium for G_5 if, and only if, $p = q > 0$ and $d = (d_1,d_2)$ satisfies $d_1 + d_2 = 1$, $d_2 \geqq \varepsilon/(2 + 2\varepsilon)$. The strategies played at a competitive stock market equilibrium are $x = y = k + 1$, leading to the pair of payoffs $(2k + 2, 2k + 2)$. At a minimum-trade equilibrium, each player holds a fraction of size $\varepsilon/(2 + 2\varepsilon)$ in the shares of the other player's payoff function.

<u>Example 6</u>. Consider two identical firms, with labor as their only variable input. When employing a labor force of size L, each firm produces a quantity of output $f(L)$. The production function f is assumed to be increasing and concave and to satisfy $f(0) = 0$ and $f(1) = 1$. The labor market is almost-competitive, in the sense that the labor force tends to flow almost entirely to the firm which offers a higher wage rate. More precisely, let $\varepsilon > 0$ be small, and let L_ε be a real continuous and non-decreasing function, defined on $[-1,1]$, such that

$$L_\varepsilon(x) + L_\varepsilon(-x) = 1 \quad \text{for all } x$$

$$L_\varepsilon(x) = 0 \quad \text{for } x \leqq -\varepsilon$$

$$L_\varepsilon(x) = 1 \quad \text{for } x \geqq \varepsilon.$$

Assume that $L_\varepsilon(w-v)$ is the fraction of the total labor force employed by a firm when it offers the wage rate w and its rival offers the wage rate v. This assumption makes the labor market "ε-competitive", tending to perfectly competitive as $\varepsilon \to 0$. Suppose that one firm offers the wage rate w, whilst its rival offers the wage rate v. This firm's payoff is then given by

$$h(w,v) = f(L_\varepsilon(w - v)) - wL_\varepsilon(w - v).$$

(For convenience, we are taking both the size of the total labor force and the price of output to be 1.) We now have a two-person game in normal form, say $\langle G_6 = X, Y, u^1, u^2 \rangle$, with $X = Y = [0,1]$ and $u^1(x,y) = u^2(y,x) = h(x,y)$ for all $(x,y) \in X \times Y$. The game G_6 depends, of course, on the value of ε, and we shall be interested in what happens as $\varepsilon \to 0$. The game G_6 has a unique symmetric Nash equilibrium, with both firms offering the same wage rate, call it $\bar{w}$, so that one-half of the labor force is employed by each firm at equilibrium. A calculation reveals, as expected, that

$$\bar{w} \sim f'(1/2)$$

where the symbol ~ is being used for equality as $\varepsilon \to 0$. Equilibrium payoffs are equal and given by

$$h(\bar{w},\bar{w}) \sim f(1/2) - f'(1/2)/2.$$

At the Nash equilibrium, as $\varepsilon \to 0$, each firm offers the competitive wage rate (i.e., the marginal product of labor) and earns the competitive payoff. Clearly, if collusion were possible, then both firms would offer the wage rate $w_o = 0$ (i.e., the monopsony wage rate) and their payoffs would be given by f(1/2) for each firm.

Now let us introduce a competitive stock market. Doing this, we find that a competitive stock market equilibrium for G_6 exists, and is characterized by any $\langle p, q, \underset{\sim}{d}, \Pi - \underset{\sim}{d} \rangle$, with $\underset{\sim}{d} = (d_1, d_2)$, such that $p = q$ and $d_1 + d_2 = 1$, $d_2 \geqq 1 - f(1/2)$. The wage rate offered by the two firms at this equilibrium is $w_o = 0$ and they both earn the monopsonistic payoff, f(1/2). At a minimum-trade equilibrium, each firm holds a fraction 1 - f(1/2) -- which can be quite small -- of its rival's shares. Competition in the labor market has been destroyed by the competition in the stock market.

4. Concluding Remark

In the foregoing discussion, the framework has been such that only two persons -- players 1 and 2 -- had access to the stock market. We would like to comment, in conclusion, on the extent to which this restriction might be crucial. Suppose that, in addition to the two players, other agents are also allowed to trade in the competitive market for shares of the two payoff functions. These "other agents", be they few or many or infinitely many, are assumed to maximize the net cash inflow from stock-market transactions (or some increasing function thereof). The two players' "utility" functions, U and V, must now be modified, to take account of these outside agents, Specifically, when player 1 contemplates an asset position, say $\underset{\sim}{d}$, he must form a conjecture on the distribution of the remaining shares between his rival and the public at large. This conjecture will determine the game $G(\underset{\sim}{d})$ and the "utility" $U(\underset{\sim}{d})$. The same holds true for player 2 and for the public at large. Stock market equilibrium may now be defined as a state in which, given the announced prices, all conjectures are fulfilled and the total supply of shares equals the sum of utility-maximizing demands. When all this is said and done, it becomes obvious that every equilibrium in the two-trader framework, with no other agents allowed, corresponds trivially to an equilibrium for the new setting, in which outside traders find it optimal to stay out of the market into which they have just been admitted. This correspondence breaks down, of course, if outside traders have reasons other than the maximization of net cash inflow (e.g. insurance) for holding shares of the two players' payoff functions.

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